Momentum, Noise Trader Risk, Experience and Experimental Asset Price Bubbles

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Abstract

This paper adapts the well-established and micro-founded model of DeLong et al. [1990b] into the framework of laboratory asset markets. The model helps to distinguish between different sources of bubble-crash patterns, usually observed in laboratory asset markets. The paper tests the theoretical predictions and estimates the model’s structural parameters, using experimental data presented in Haruvy et al. [2007], Dufwenberg et al. [2005] and Kirchler et al. [2012]. The results suggest that experimental asset price bubbles and crashes are primarily driven by the interaction between short-term price-momentum and noise-trader risk. Experience reduces bubbles in laboratory asset markets primarily through a mitigation of noise trader risk.

Keywords: Experimental Asset Markets, Bubbles, Noise Trader Risk

JEL Classifications: C90, C91, D03, G02, G12

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1 Motivation

Under the efficient market hypothesis (EMH) the price of a financial asset cannot differ systematically from its fundamental value (FV) – defined as the present value of the expected dividend stream – if there are no limits to arbitrage and traders are fully rational (Shleifer [2000]). Despite its theoretical elegance, the empirical validity of the EMH has been challenged ever since the seminal works of Shiller [1981], LeRoy and Porter [1981], Roll [1984], De Bondt and Thaler [1985], Mehra and Prescott [1985] and Lakonishok et al. [1991, 1994]. Since then the literature identified various sources for limited arbitrage-opportunities, in conjunction with behavioral biases, which may together contribute to the frequent rejection of the EMH in field data.¹ But the rejection of the EMH prevails even in stylized and controlled experimental environments.

In their seminal paper, Smith et al. [1988] (SSW) report that experimental asset prices may deviate systematically from the underlying fundamental value process, even if the dividend distribution is common knowledge. Moreover, SSW show that laboratory asset prices do not simply deviate from the underlying FV but follow paths which can best be described as bubble-crash patterns: Initially, asset prices increase well beyond the fundamental value (bullish phase) until they peak and “crash” drastically (bearish phase). SSW also show that experience – achieved via repeated participation – mitigates bubbles substantially (see also e.g.: Dufwenberg et al. [2005], Haruvy et al. [2007], Hussam et al. [2008]).

Bubble-crash patterns with inexperienced subjects proved to be highly replicable and persist under various experimental treatments.² Some authors assert that experimental asset price bubbles may emerge from the interaction of speculators and trend-chasers (e.g., Smith et al. [1988], Caginalp and Ilieva [2005], Ackert et al. [2006], Haruvy and Noussair [2006], Moinas and Pouget [2012], Baghestanian et al. [2012]), while others suggest that the observed price-patterns may be driven by “confusion” (e.g., Lei et al. [2001], Lei and Vesely [2009], Smith [2010], Oechsler [2010], Oechsler et al. [2011], Kirchler and Huber [2011], Kirchler et al. [2012]).

In other words, a clear understanding of the factors leading to the formation of bubbles and crashes in laboratory environments is still missing. More specifically, we are lacking a coherent micro-founded equilibrium model which generates accurate price predictions for experimental asset markets. Furthermore, we do not yet fully understand the mechanism through which experience reduces bubbles in laboratory environments.

This paper presents a concise model which shows that noise trader risk is an important ingredient for the formation of experimental asset price bubbles and crashes. Moreover, it will be shown that experience mitigates bubbles in laboratory asset markets via a reduction in noise trader risk.

Risk from noise trading arises if rational investors think that beliefs of noise traders may become more extreme or more optimistic in the short run before they revert back to the mean (DeLong et al. [1990b]). In other words, if rational investors believe that mis-pricing may deepen in the short run, they may have an incentive to buy an overpriced asset, so that they can sell it to a “greater fool”, who is willing to pay an even higher price, in the following period. The presence of noise trader risk and systematic noise-trading in general has been widely accepted in the behavioral and empirical finance literature.³ However, an investigation of the effects of noise trader risk on the formation of asset price bubbles in experimental trading environments is still missing.

To fill this gap I adapt the model of DeLong et al. [1990b] (DSSW) into the framework of experimental asset markets. The finite-horizon model consists of noise traders and arbitrageurs, who decide over their

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¹For a detailed review on the factors, which contribute to the partial empirical falsification of the EMH see Shleifer [2000]. One branch in the behavioral finance literature shows that asset prices, well above the underlying FV, can be sustained for prolonged periods (bubbles), before they may potentially revert back (crash) to the assets fundamental value. See e.g.: Harrison and Kreps [1978], Tirole [1982], Hart and Kreps [1986], DeLong et al. [1990b], Cutler et al. [1990], Hirshleifer [2001], Abreu and Brunnermeier [2003], Brunnermeier and Pedersen [2005].

²See e.g.: King et al. [1993], Lei et al. [2001], Boening et al. [1993], Friedman [1993], Smith et al. [2000], Caginalp et al. [2000], Noussair and Tucker [2006], Haruvy and Noussair [2006], Haruvy et al. [2007], Noussair et al. [2001], Kirchler and Huber [2011].

³See e.g.: Black [1986], Daniel et al. [1998], Shiller [2000], Hirshleifer [2001], Lamont and Thaler [2003], Barber et al. [2009].
optimal demands for a risky asset. Along the lines of DSSW, both types have short foresight and maximize their next period’s wealth under standard mean-variance utility functions. Noise traders differ from arbitrageurs by making systematic and optimistic forecasting errors, which follow stationary autoregressive processes. I derive the equilibrium price process and discuss its theoretical properties. I will show that the resulting equilibrium price path can be decomposed into a higher-order-belief term (greater fool term), a risk-compensation term and the fundamental value of the asset. The greater-fool term generates positive deviations from the assets’ fundamental value and is responsible for the emergence of bubbles in the model. The risk-compensation term generates negative pressure on prices: Once the risk compensation term outweighs the greater fool term the bubble bursts and prices crash towards the FV.

The model generates other interesting insights. It will be shown that noise-trading strategies in experimental asset markets may be more profitable than strategies adopted by more rational arbitrageurs. This insight supports the results in Lei et al. [2001] and Kirchler et al. [2012], suggesting that “confusion” is a persistent phenomenon in laboratory asset markets.

In a next step I extend the baseline model by introducing various type-transition dynamics. That is, I investigate the resulting price dynamics if agents are allowed to transition from one type to the other over the course of the entire trading horizon. It will be shown that introducing these, potentially history dependent, transition dynamics generates pronounced bubbles and crashes in the model, which also fit standard experimental asset market data.

Furthermore, introducing these transition dynamics helps to distinguish between three, potentially different, sources of bubbles and crashes in laboratory environments: Increasingly rational behavior, increasingly irrational behavior and noise trader risk arising from price momentum. I apply the theoretical insights from the model to identify the sources of the bubble-crash patterns reported in Haruvy et al. [2007] (henceforth HLN) for inexperienced and experienced subjects. The results suggest that the main source for the price-swings reported by HLN is noise trader risk arising from price-momentum. The empirical findings further indicate that experience reduces bubbles through a mitigation of noise-trader risk. The robustness of these results is tested via additional data-sets, presented in Dufwenberg et al. [2005] and Kirchler et al. [2012]. In the discussion-section I will show that the model can also be used to explain various other recent findings reported in the literature, such as the results in Eckel and Füllbrunn [2013], Cheung et al. [2014] and Baghestanian and Walker [2014].

By introducing type-transition dynamics into the baseline model new insights into the mechanics of experimental asset price bubbles are generated: Price rallies in laboratory environments are primarily driven by an increasing number of optimistic noise traders, potentially due to momentum, initially sparked by heterogeneous price expectations. Once a certain threshold-level of irrational noise traders is surpassed, their positive price effect is outweighed by their positive impact on the noise-trader-risk-compensation-premium, charged by arbitrageurs. Put differently, if the number of noise traders with excessively optimistic price expectations becomes sufficiently large, the risk that they are subject to negative mood-swings increases as well. Arbitrageurs take this potential risk into account, discount the asset’s value accordingly as the trading horizon narrows and induce a crash.

In other words, bubbles and crashes in experimental asset markets are not the result of “confusion” or “speculation” alone but rather emerge from the interplay of “confusion” – formalized via noise trading – and rational behavior. The results further indicate that experience increases the speed of type transitioning, decreases noise trader risk and thereby mitigates bubbles.

The main contribution of this paper is that it identifies momentum, noise-trader risk and type-transitioning as important factors, which contribute to the formation of laboratory asset price bubbles and crashes. Relative to Duffy and Ünver [2006], who present an agent-based model with noise traders to generate bubble-crash patterns, the agents in the model presented in this paper behave optimally relative to their short foresight-horizons. Moreover, in contrast to Duffy and Ünver [2006], the model admits heterogeneous trading behavior. Haruvy and Noussair [2006] also consider heterogeneous trading behavior and provide an adapted agent-based model to explain the emergence of bubbles and crashes in experimental asset markets. This evidence is consistent with the results in Akiyama et al. [2013] and Cheung et al. [2014].

Baghestanian et al. [2012] also consider an agent based model to explain the emergence of bubbles and crashes in experi-
version of the model of DeLong et al. [1990a]. While their work investigates the effect of feedback-trading on the formation of experimental asset price bubbles, I focus on the impact of noise-trader risk. Moreover, the model presented in this paper allows type-transitioning and investigates the mechanism through which experience reduces bubbles.\textsuperscript{6} From a more general perspective, the paper aims to further connect the insights from the theoretical- and the experimental finance literature.

2 The Model

Along the lines of DeLong et al. [1990b], I consider an environment in which a continuum of traders, evenly distributed on the unit interval, trade a single financial asset over 1,\ldots,\(T\) periods. The assumption of a “large” set of traders abstracts from the possibility of strategic price manipulation and makes a focused investigation of the relationship between noise trader risk and laboratory asset price bubbles possible.\textsuperscript{7}

Along the lines of DSSW and in accordance with standard experimental asset market designs, the financial asset is assumed to be in fixed supply. In contrast to DSSW, the asset does not pay a fixed but a random dividend, \(d_t\), at the end of every period. For the sake of simplicity, I assume that the distribution of \(d_t\) can be approximated by a normal distribution with mean \(\bar{d}\) and variance \(\sigma_d^2\).\textsuperscript{8} Moreover, the dividends are assumed to be independently distributed over time. Following the experimental literature, I will denote with \(FV_t\) the fundamental value of the asset in period \(t\) under risk neutrality, homogeneity and rationality, i.e.:

\[
FV_t = \bar{d}(T - t + 1) \quad t = 1,\ldots,T.
\]

The model consists of a measure of \(1 - \mu\) arbitrageurs (\(a\)) and a measure of \(\mu\) noise traders (\(n\)). Both types of traders receive an initial endowment of \(w_0\) in period zero, which can be thought of as the cash endowment given in standard laboratory asset market environments. For the sake of convenience, I assume that this initial cash endowment is sufficiently large to satisfy any feasible demand for the risky asset at any finite price. Further, both traders maximize utility functions of the form:

\[
U(w_{t+1}) = -e^{-2\delta w_{t+1}},
\]

where \(\delta > 0\) is the coefficient of absolute risk aversion and \(w_{t+1}\) is an agent’s wealth in period \(t + 1\). Arbitrageurs and noise traders maximize their expected utilities over short horizons\textsuperscript{9} and decide in every period how much of their wealth they want to invest into the risky asset.

The quantities of the risky asset, demanded by arbitrageurs and noise traders, are denoted with \(\lambda^a_t\) and \(\lambda^n_t\) respectively. The remaining portions of their portfolios, which are not invested into the risky asset, are denoted with \(S^a_t\) and \(S^n_t\). Furthermore, the price of the risky asset in period \(t\) is given by \(p_t\).

In standard experimental asset markets risk-free assets, which pay a non-zero return, are usually not provided to subjects.\textsuperscript{10} Hence, I will assume that the risk-free rate is normalized to zero. The wealth dynamics between periods \(t\) and \(t + 1\) are therefore described by:

\[
w^a_t = S^a_t + \lambda^a_t p_t \quad w^n_t = S^n_t + \lambda^n_t (p_{t+1} + d_{t+1}) \Rightarrow w^i_{t+1} = w^i_t + \lambda^i_t (p_{t+1} + d_{t+1} - p_t). \quad i \in \{a, n\}
\]

\textsuperscript{6}Similarly, Caginalp and Ilieva [2005] and Caginalp and Merdan [2007] also present heterogeneous agent models to explain bubble-crash patterns in experimental asset markets. In their models traders do not transition from one type to another. Moreover, crashes in their models are generated by binding cash-constraints and not via the interplay of a narrowing trading horizon and noise-trader risk.

\textsuperscript{7}This assumption is made since the main purpose of this paper is to focus on the impact of noise-trader risk on bubble formation.

\textsuperscript{8}This approximation is innocuous for the quantitative analysis below but allows for closed form solutions in the theory section. In the numerical analysis below \(\sigma^2_d\) is chosen to be sufficiently large so that the corresponding normal distribution (if appropriately truncated) is close to a uniform distribution over the relevant subset of the dividend support.

\textsuperscript{9}I.e.: they focus on the value of their portfolios in period \(t + 1\).

\textsuperscript{10}One of the few exceptions are Bossaerts et al. [2007] and Fischbacher et al. [2013].
Lastly, I assume that arbitrageurs form rational expectations in every period and that noise traders in period \( t \) misperceive the expected price of the risky asset in period \( t + 1 \) by a random variable \( \rho_t \):

\[
\rho_t = \eta + \gamma \rho_{t-1} + \epsilon_t
\]

where \( \eta > 0 \), \( \gamma \in [0, 1) \) and \( \epsilon_t \) is an independent and identically distributed normal random variable with mean zero and variance \( \sigma^2 \). The variable \( \rho_t \) represents the period-\( t \) “bullishness” of noise traders, who are subject to mood swings as measured via the random variable \( \epsilon_t \). The variance of mood-swings, \( \sigma^2 \), formalizes the risk induced by noise-traders, which will have important effects on the underlying equilibrium price dynamics.

I assume that mood-swings, \( \epsilon_t \), and dividends, \( d_t \), are independently distributed across periods. Given these assumptions the distribution of wealth in period \( t + 1 \) is given by

\[
-2\delta w_{t+1}^i \sim N(-2\delta \mathbb{E}_t(w_{t+1}^i), 4\delta^2 \text{Var}_t(w_{t+1}^i)), \quad i \in \{a, n\}
\]

where \( \mathbb{E}_t \) and \( \text{Var}_t \) denote the expectations and variance operators, conditional on all the information available up to period \( t \). \( N(a, b) \) denotes the normal distribution with mean \( a \) and variance \( b \). Hence, maximizing the conditional expectation of \( U(w_{t+1}) \), as specified above, is equivalent to minimizing (1) with respect to \( \lambda^i_t \) with \( i \in \{a, n\} \).

\[
- \delta \mathbb{E}_t(w_{t+1}^i) + \delta^2 \text{Var}_t(w_{t+1}^i), \quad i \in \{a, n\}
\]

Letting

\[
\sigma^2_{t+1,p+d} = \text{Var}_t(d_{t+1} + p_{t+1}) = \mathbb{E}_t(d_{t+1} + p_{t+1} - \bar{d} - \mathbb{E}_t(p_{t+1}))^2,
\]

the resulting demand functions for arbitrageurs and noise traders are given by:

\[
\lambda^a_t = \frac{\bar{d} + \mathbb{E}_t(p_{t+1}) - p_t}{2\delta \sigma^2_{t+1,p+d}}, \quad \lambda^0_t = \frac{\bar{d} + \mathbb{E}_t(p_{t+1}) - p_t + \rho_t}{2\delta \sigma^2_{t+1,p+d}}.
\]

For the sake of simplicity I assume that the asset is in unit supply, to solve for the equilibrium price path. The unit-supply assumption is innocuous and does not affect the following results qualitatively, as long as the asset is in fixed and constant supply.\(^{11}\) In summary –before I proceed to the characterization of the equilibrium– the simple model described above deviates from the baseline model of DSSW, by assuming that the trading horizon is finite; dividends are random; bullishness follows an autoregressive process; the risk free rate is normalized to zero.

Based on the assumptions given above, the period \( t \) equilibrium price process is given by:

\[
p_t = \bar{d} + \mathbb{E}_t(p_{t+1}) - 2\delta \sigma^2_{t+1,p+d} + \mu p_t.
\]

Using the terminal condition that \( \mathbb{E}_T(p_{T+1}) = 0 \), the equilibrium price path in (2) can be solved recursively to obtain:\(^ {12}\)

\(^{11}\) If the asset would be in random supply, the model would admit multiple equilibria (Walker and Whiteman [2007], Branch and Evans [2011]).

\(^{12}\) Recursive steps: I denote with \( \sigma = \sigma^2_{t+1,p+d} \) and show later above that \( \sigma^2_{t+1,p+d} \) can be approximated by a time-invariant expression:

In Period \( T \):

\[
p_T = \bar{d} - 2\delta \sigma \mu p_T
\]

In Period \( T - 1 \):

\[
p_{T-1} = \bar{d} - 2\delta \sigma \mu p_{T-1} + \bar{d} - 2\delta \sigma \mu \mathbb{E}_{T-1} p_{T-1} = \bar{d} - 2\delta \sigma \mu p_{T-1} + \bar{d} - 2\delta \sigma \mu (\eta + \gamma p_{T-1}) = 2\bar{d} - 2(2\delta \sigma) + \mu \eta + \mu (1 + \gamma) p_{T-1}
\]
In period \( T - 2 \)

\[
p_{T-2} = 3\bar{d} - 3(2\delta \sigma) + \mu \rho_{T-2} + \mu \eta + \mu (1 + \gamma) \overline{\eta}_{T-2} \rho_{T-1} =
3\bar{d} - 3(2\delta \sigma) + \mu \rho_{T-2} + \mu \eta + \mu (1 + \gamma) + \mu (1 + \gamma + \gamma^2) \rho_{T-1}
\]

In Period \( T - 3 \)

\[
p_{T-3} = 4\bar{d} - 4(2\delta) + \mu \eta (1 + (1 + \gamma) + (1 + \gamma + \gamma^2)) + \mu (1 + \gamma + \gamma^2) \rho_{T-3}
\]

... and so forth. Noticing that \((T - t + 1)d = FV_t\) gives the desired result.
experimental asset markets— even if the dividend-volatility ($\sigma_d^2$) would degenerate at zero as in Porter and Smith [1995]. The results support the conclusion in Porter and Smith [1995] suggesting that the initial negative deviation from the asset’s FV is not driven by risk aversion associated with dividend uncertainty. Instead, the model suggests that the initial negative deviation could be generated by aversion against noise trader risk, jointly measured via $\delta \frac{\mu^2 \sigma_d^2}{(1-\gamma)^2}$ even if the dividend uncertainty was zero.

- The third term in (5) is the “greater fool” term. It captures a price-pressure effect, induced by bullish noise traders. In general, this sum of sums explodes if $T \rightarrow \infty$, but since the trading horizon is finite, the bullishness-term decreases over time. The decrease captures the awareness of arbitrageurs that they may benefit more from the average bullishness of noise traders, the longer the remaining trading horizon. Note that the price-pressure effect corresponds to the usual “bubble term” in infinite horizon models, under the presence of a risk free trading option. In the standard infinite horizon framework “bubble terms” are eliminated via transversality conditions. Furthermore, the size of the bubble term depends positively on the fraction of noise traders, their systematic bullishness $\eta$ and the persistence of their mood swings $\gamma$.

- The last term in (5) captures the accumulated impact of shocks to bullishness on equilibrium prices in period $t$.

Similar to DSSW, I next investigate whether noise trading in experimental asset markets could in principle be profitable. If noise trading leads to elevated returns, then the evolutionary argument provided by Friedman [1953] and Fama [1965], which suggests that noise traders disappear from financial markets due to the losses they incur, would not necessarily hold for experimental asset markets. Along the lines of DSSW, I compute the unconditional expected difference between the returns of noise traders, $R^n_t$, and arbitrageurs, $R^a_t$ as:

$$\mathbb{E}(\Delta R_t) = \mathbb{E}(R^n_t - R^a_t) = \mathbb{E}(d_{t+1} + p_{t+1} - p_t)(\lambda^n_t - \lambda^a_t) \approx \mathbb{E}(d_{t+1} + \tilde{p}_{t+1} - \tilde{p}_t)(\lambda^n_t - \lambda^a_t).$$

I show in Appendix A that the approximated difference in (7) is zero whenever

$$\frac{2\delta \sigma_{t+1,p,d}^2 \gamma}{1-\gamma} - \mu \gamma^2 \frac{1-\gamma^{T-t+1}}{(1-\gamma)^2} - \mu \gamma^2 \frac{1-\gamma^T}{(1-\gamma)^2} - \mu \gamma^2 \frac{1-\gamma^T-\gamma^T}{(1-\gamma)^2} = 0,$$

where again:

$$\sigma_{t+1,p,d}^2 \approx \sigma_d^2 + \frac{\mu^2 \sigma_t^2}{(1-\gamma)^2}.$$

In words, whenever the moment condition in (8) holds, trading strategies of arbitrageurs lead to the same returns as strategies used by noise traders. The following lemma summarizes under what conditions optimistic noise trading may be more profitable than arbitraging in experimental asset markets.

**Lemma 2.1.** If $\sigma_d^2 = 0$, there exists a unique $\mu_t^*$ for every $t$ given by:

$$\mu_t^* = \frac{\eta^2 (1-\gamma^{T-t+1})(1-\gamma) + (1-\gamma)^2 + \gamma^{T-t} \eta^2 (1-\gamma)}{2\delta \eta},$$

so that if $\mu_t > \min(\mu_t^*, 1)$, all noise traders gain higher returns than arbitrageurs in period $t$.

If $\sigma_d^2 > 0$ and

$$\sigma_d^2 < (1-\gamma) \frac{\eta^2 (1-\gamma^{T-t+1})(1-\gamma) + \sigma_t^2 (1-\gamma)^2 + \gamma^{T-t} \sigma_t^2 (1-\gamma)}{2\delta \eta} = \sigma_d^*,$$
there exists a unique $\mu_t^* \in (0, 1)$ for every $t$ so that if $\mu_t \in [0, \mu_t^*)$, all noise traders gain higher returns than arbitrageurs in period $t$. If $\mu_t \in (\mu_t^*, 1]$, all arbitrageurs gain higher returns than noise traders.

If $\sigma_2^2 > \sigma_1^2$ there are three possibilities: 1.) Noise traders always gain higher returns than arbitrageurs in every period $t$. 2.) In some periods there exist two values, $(\mu_{1,t}^*, \mu_{2,t}^*)$, so that noise traders may gain higher returns in period $t$, if $\mu_t < \mu_{1,t}^*$ or $\mu_t > \mu_{2,t}^*$. 3.) In some periods there exists a unique $\mu_t^*$, so that noise traders make higher returns if $\mu_t \in [0, \mu_t^*)$ and lower returns than arbitrageurs if $\mu_t \in (\mu_t^*, 1]$ in period $t$.

In summary, lemma 2.1 shows that noise trading could be profitable in experimental asset markets and may therefore be a persistent phenomenon as suggested in Lei et al. [2001] and Kirchler et al. [2012]. The first part of the lemma shows that noise-traders may persist even in environments in which dividend uncertainty is eliminated as in Porter and Smith [1995]. Moreover, it also clear that the parabola in (7) would always be positive if $\sigma_2^2$ is sufficiently large. Hence, if the the fundamental risk is high, noise trading is always more profitable than arbitraging in laboratory asset markets. In general this implies that noise traders may not only persist in asset market experiments but may also make higher returns than more rational arbitrageurs.

3 Sources of Bubbles and Crashes in Experimental Asset Markets

Although the equilibrium price path in (5) explains why experimental asset prices may deviate systematically from the underlying fundamental value, the resulting equilibrium price dynamics are not entirely consistent with the patterns usually observed in the data. That is, the baseline formulation of the model does not permit sharp price-rallies followed by sudden crashes. To see this, note that the unconditional expectation of price-changes are given by:

$$E\Delta p_{t+1} = E(\bar{p}_{t+1} - \bar{p}_t) = 2\sigma_d^2 \delta + \frac{2\delta \mu^2 \sigma_e^2}{(1 - \gamma)^2} - \frac{\mu\eta}{1 - \gamma} + \frac{\mu\eta}{1 - \gamma} \left(\gamma^{T-t+1} - \gamma^{T-t}\right).$$

(9)

Although the sign of the expression in (9) may generally change from positive to negative over the trading horizon, it is clear that the sign-transition tends to be rather smooth.\(^{13}\)

These smooth price patterns, which do not fit standard experimental asset market data, emerge under the strict assumption that type distributions remain unchanged over the entire trading horizon. However, this assumption is fairly implausible. Future beliefs about asset prices—which determine an agent’s trader-type in the model—depend crucially on experience and realized price paths. As prices change, traders may become more or less optimistic and therefore adapt their trading behavior.

It is therefore generally desirable to incorporate type transitions into the model. Moreover, I will show that introducing these additional dynamics generates pronounced bubbles and crashes in the model, which also fit standard experimental data. More importantly, as I will show below, introducing the possibility that agents may change type helps identifying different sources for bubbles and crashes in experimental data.

To incorporate type transitions into the model, I consider evolutionary dynamics similar to the ones described in DeLong et al. [1990b]. To be specific, I consider type-evolutions which have the following functional form:

$$\mu_{t+1} = \min\left\{\max\left\{0, \mu_t \pm \phi_f(F^t)\right\}, 1\right\} \quad t = 1, ..., T,$$

(10)

where $f(F^t)$ is a function, mapping elements from the information set $F^t$ to the real numbers and $\phi > 0$ is a parameter measuring the speed of the type-transition. Further details on the specification of $f(F^t)$ are provided below.

Introducing evolutionary dynamics into the model above, may generally make the equilibrium price analysis more complicated, since rational arbitrageurs would always want to condition their behavior on the expected type-evolution. There are two methods to circumvent the associated complications.

\(^{13}\)Note that $\frac{\partial \Delta p_{t+1}}{\partial t} = \frac{2\mu}{1 - \gamma}(-e^{(T-t+1)\ln \gamma} + e^{(T-t)\ln \gamma})$. 
First—following the suggestion of DSSW— if $\phi \to 0$, the equilibrium price process, which emerges under type-evolution-dynamics given in (10), can be approximated via:

$$\hat{p}_t = FV_t - 2\delta(T - t + 1)(\sigma^2_d + \mu^2_t \sigma^2_e (1 - \gamma)^2) + \mu_t \eta \sum_{k=0}^{T-t} \sum_{j=0}^k \gamma^j + \mu_t \rho \sum_{j=0}^{T-t} \gamma^j, \quad (11)$$

In other words, if the type evolution is sufficiently slow, prices in (11) provide accurate approximations of equilibrium prices in the model above under (10).

Secondly, one could motivate the approximation in (11) assuming that arbitrageurs hold false beliefs about the true type evolution. For instance, if arbitrageurs falsely believe that:

$$E_t(\mu_{t+j}) = \mu_t \quad \forall \ j \geq 1,$$

either because the type evolution dynamics are not common knowledge or because arbitrageurs cannot accurately predict the future fraction of noise traders, the equilibrium price process in (11) correctly approximates prices in the model above, under type dynamics given in (10).

The assumption of false beliefs about future fractions of noise traders seems to be particularly accurate for experimental closed book call market environments, in which the financial asset trades at a uniform market clearing price and subjects do not observe the individual demands of other participants. I will therefore stick to the more plausible “false belief” assumption to motivate the equilibrium price prediction in (11).

I investigate the implied price-properties of three important evolutionary type-transition dynamics. Under the first pattern, I assume that agents overall behave increasingly rational and have increasingly accurate price-forecasts. In other words, the fraction of noise traders decreases over time according to the exogenously given learning rule:

$$\mu_{t+1} = \min \left\{ \max \left\{ 0, \mu_t - \phi(t-1) \right\}, 1 \right\}, \quad t = 1, \ldots, T. \quad (12)$$

The dynamics given in (12) are important to investigate since it might be reasonable to assert that subjects learn over time and behave increasingly rational. The underlying linear functional form is chosen for simplicity purposes and serves as an approximation for other monotonic learning functions. Since the fraction of arbitrageurs, who condition their behavior on noise traders increases under the dynamics given in (12),
I will refer to them as higher-order-belief dynamics (HOBD), emphasizing that the notion of higher order beliefs in this context differs significantly from the notion of higher-order beliefs usually used in the (e.g.) macro-literature.\textsuperscript{14}

Under the second pattern I assume that agents behave increasingly noisy/optimistic. These particular dynamics are investigated due to experimental evidence suggesting that individual price-expectations of participants in laboratory asset markets become more homogeneous over time but do not necessarily converge to rational expectations (Hommes et al. [2005, 2008] and Hommes [2010]). To be specific, pattern two assumes that the fraction of noise traders increases over time according to the exogenously given rule:

$$\mu_{t+1} = \min \left\{ \max\left\{ 0, \mu_t + \phi(t-1) \right\}, 1 \right\}, \quad t = 1, ..., T. \quad (13)$$

Since the fraction of irrational traders increases under the dynamics given in (13), I will refer to them as confusion dynamics (CD).

Lastly, under the third pattern, I consider evolutionary dynamics which depend on short term “momentum”. In particular, I assume that short term price increases lead to increased optimism and short term price drops lead to decreased optimism and increasingly rational behavior. In other words, under pattern three, the fraction of noise traders evolves over time according to the exogenously given rule:

$$\mu_{t+1} = \min \left\{ \max\left\{ 0, \mu_t + \phi\left(\frac{p_t-1-p_{t-2}}{t}\right) \right\}, 1 \right\}, \quad t = 3, ..., T. \quad (14)$$

I will further refer to the dynamics given in (14) as short-term-momentum-dynamics (STMD).\textsuperscript{15}

The three type-dynamics given above have interesting properties. First of all, all three of them can generate bubbles and crashes in the model under appropriate parameterizations. To see this, note that the expression in (9) is non-linear in \( \mu_t \) and may therefore admit two positive roots under appropriate parameter-choices. It is therefore possible to identify non-empty parameter-regions in which high values of \( \mu_t \) induce price increases whereas low values induce a drop in prices and vice versa-- i.e.: it is possible that bubble-crash patterns emerge under HOBD and CD.

Under STMD, an increase in prices from period \( t-2 \) to period \( t-1 \) leads to an increase in noise trading. Looking at the risk-compensation term in (11) (second term), it is clear that a sufficient increase in noise traders may actually result into a drop of prices in period \( t \). That is, the more optimists arrive on the market, the higher the risk for arbitrageurs that a large number of traders is subject to the same mood-swings. To insure themselves against negative mood-shocks arbitrageurs essentially “demand” a reduction of prices before they are willing to hold the asset in equilibrium. If this risk-compensation premium outweighs the higher-order belief terms in (11),\textsuperscript{16} prices start to fall. Looking at (14), the drop in prices in period \( t \) could result into a drastic erosion of noise traders in period \( t+1 \). As a matter of fact, the fraction of noise traders could even drop all the way down to zero, in which case the equilibrium prices would crash down to the risk-adjusted fundamental value of the asset given in (6).

I illustrate several simulated price paths under STMD in Figures 1 and 2. The purpose of these simulations is to show that the model admits price paths, similar to the ones usually observed in experimental asset markets. For all the simulations I let \( \bar{d} = 24 \) and \( T = 15 \). Hence, the risk neutral rational expectations equilibrium prices, represented by the straight and solid lines in Figures 1a to 2b, are equal to 360 in period one and decrease by a value of 24 in every period. Similar dividend- and time-horizon parameters are used in experiments by Baghestanian et al. [2012], Eckel and Füllbrunn [2013] and Haruvy et al. [2007]\textsuperscript{17}. The

\textsuperscript{14}In a standard higher order beliefs context, noise traders and arbitrageurs do not simply differ in terms of the average misperception of noise traders but in terms of the underlying information sets (e.g.: Bacchetta and van Wincoop [2004], Walker and Rondina [2010]). Introducing different information sets into the environment above, would not enable me any longer to determine the equilibrium price process as above. The recursive formulation would become increasingly complicated due to the failure of the law of iterated expectations (Allen et al. [2006]).

\textsuperscript{15}I will further assume that under STMD, for the first two periods, the type dynamics follow CD with the same \( \phi \)-specification as under STMD.

\textsuperscript{16}Terms 3 and 4.

\textsuperscript{17}HLN use \( \bar{d} = 12 \).
Figure 2: Simulated Price Paths under STMD II.

(a) Straight line: $FV_t$. Parameters for dashed line without markers: 
$\sigma^2 = 0.17, \gamma = 0.77, \mu_0 = 0.3, \delta = 0.4, \eta = 82, \bar{d} = 24, \sigma^2_d = 2.3, \phi = 0.0036.$

(b) Straight line: $FV_t$. Parameters for dashed line without markers: 
$\sigma^2 = 0.17, \gamma = 0.78, \mu_0 = 0.9, \delta = 0.47, \eta = 81, \bar{d} = 24, \sigma^2_d = 8.3, \phi = 0.0036.$

Figure 3: Simulated Price Paths under HOBD (Right Panel) and CD (Left Panel).

(a) Straight line: $FV_t$. Parameters for lowest line: $\sigma^2 = 4.07, \gamma = 0.835, \mu_0 = 0.1, \delta = 0.847, \eta = 41, \bar{d} = 24, \sigma^2_d = 11, \phi = 0.003$. Parameters for middle line: $\delta = 0.84$ - all other parameters unchanged. Parameters for highest line: $\delta = 0.83$ - all other parameters unchanged.

(b) Straight line: $FV_t$. Parameters for lowest line: $\sigma^2 = 3.96, \gamma = 0.972, \mu_0 = 0.92, \delta = 0.3542, \eta = 258.9, \bar{d} = 24, \sigma^2_d = 4, \phi = 0.0009$. Parameters for middle line: $\delta = 0.25$ - all other parameters unchanged. Parameters for highest line: $\delta = 0.2$ - all other parameters unchanged.
values of the other parameters are given underneath the corresponding plots. Most importantly, in all simulations the value of $\phi$ is chosen to be fairly small, to ensure that the prices given in (11) approximate the prices in the baseline model under STMD sufficiently well.

Figure 1a shows price paths, with extremely pronounced bubbles, whereas Figure 1b shows price paths which are fairly consistent with price paths usually observed in laboratory asset markets. Figures 2a and 2b illustrate that the model does not necessarily generate bubbles and crashes under STMD under appropriate parameterizations.

Figures 3a and 3b show simulated price paths under CD and HOBD respectively, with varying levels of risk aversion. Under both type-transition-dynamics, decreased risk aversions lead to higher prices. Clearly, under HOBD, as $\mu_t \to 0$ prices converge towards the risk adjusted FV of the asset. Under CD, prices converge to levels, which exceed the FV of the asset.

In summary, HOBD, CD and STMD may generate price patterns, which are consistent with standard bubble-crash patterns usually observed in experimental asset markets. Nevertheless, the underlying mechanisms, which are responsible for the observed price paths differ substantially. In the next section, I will use the three type-transition dynamics to identify the roots for bubbles and crashes in experimental data.

I will test the following hypotheses:

**Hypothesis 3.1 (HOBD).** $H_0$: Bubbles and crashes in experimental asset markets are generated by HOBD.

**Hypothesis 3.2 (CD).** $H_0$: Bubbles and crashes in experimental asset markets are generated by CD.

**Hypothesis 3.3 (STMD).** $H_0$: Bubbles and crashes in experimental asset markets are generated by STMD.

4 Noise Trader Risk and Experimental Data

This section tests hypotheses 3.1 - 3.3, using the data of Haruvy et al. [2007] (HLN).

HLN analyze laboratory asset markets consisting of eight to nine participants trading a single financial asset. The dividends of the asset were drawn from a uniform distribution with support $\{0, 4, 14, 30\}$ in every period\(^{18}\) and were denominated in an experimental currency. Since the asset paid on average 12 units of the experimental currency, the fundamental value of the asset, under homogeneity, rationality and risk neutrality, was 180 in the first period and decreased to 12 in period 15 (note that dividend payments were deferred to the end of the period). In each of the six experimental sessions, subjects were asked to participate in three to four repeated trading environments (labelled as markets 1 - 4), each consisting of fifteen trading rounds. Hence, subjects in market 1, are inexperienced, subjects in market 2 are once-experienced and so forth.

The authors use a call market trading institution to determine uniform market clearing prices in every period.\(^{19}\) Furthermore, during each of the six experimental sessions, subjects were asked to submit their price expectations about the following market clearing prices (all follow up periods), at the beginning of every trading round.

I replicate the price paths (scatter points) and the fundamental value processes (straight lines) reported by HLN in Figures 8a - 8d in Appendix A. In accordance with results previously reported in the literature, the authors report that inexperienced subjects (market 1), generate bubbles and crashes in laboratory asset markets. An increase in experience tends to mitigate bubbles (markets 2 - 4).

To test the hypotheses from above and to investigate the sources for bubbles and crashes in experimental asset markets, I initially focus on the individual price expectations data. For each individual, I first compute the percentage difference between expected and realized prices in period $t$. In accordance with the model, I

\(^{18}\)They were independently distributed over time.

\(^{19}\)Traders submit limit orders (Bids or Asks and corresponding quantities), which are ordered by the auctioneer. Bids (price element of the demand schedules) are ordered from highest to lowest to obtain the inverse demand schedule, while asks (price element of the supply schedules) are ordered from lowest to highest to obtain the inverse supply schedule.
focus on period-\(t\) price expectations given at the beginning of period \(t\) - i.e.: I focus on price expectations provided by participants, conditional on period-\(t-1\) information:

\[
\Delta E^{p}_{p,p,t} = \left| E^{i}_{t-1}p_t - p_t \right| \frac{p_t}{p_t}.
\]  

(15)

Note that both hypothesis 3.1 and 3.2 imply that (15) has to decrease over time. To see this note that if there are primarily (e.g) noise traders on the market, the generated equilibrium price will be close to the price expectation of noise traders. Hence noise traders will have better (worse) price forecasts if there are more (less) noise traders on the market. In other words if traders would homogenize over time –as implied by hypotheses 3.1 and 3.2– prices would tend to converge to the corresponding majority-type price forecast and (15) would decrease over time. Put differently, monotonic changes in the type composition will always result into better price forecasts for the average subject.

To investigate whether there are monotonic trader-type evolution patterns in the data of Haruvy et al. [2007] I classify subjects based on their price forecast accuracy in (15). I use the incentive structure employed in the experiments of HLN to classify each individual into one of the two types in the model. The incentive structure used by HLN was as follows: Whenever subjects generated price forecasts for period \(t\) within 10% of the actual realized prices of period \(t\) they received the largest financial reward, followed by the second largest reward whenever the forecast was within a 25% accuracy-band, followed by the third largest financial reward whenever the forecast was within 50% of the actual price. Subjects who provided forecasts which deviated by more than 50% from the actual price received no financial compensation.

I start by using the 10% accuracy level and compute the fraction of subjects with period-\(t\) price-forecast which differed by more than 10% from the actual period \(t\) price for every period and market. The evolution of the forecast-deviations for every period and market are shown in Figure 4a. The evolution of forecast-deviations using the 25% and 50% cutoff levels are shown in Figures 4b and 4c respectively.

By inspecting Figure 4 some interesting patterns can be observed: First, there tends to be a fairly large number of subjects with fairly inaccurate price forecasts in almost every period independently of the chosen cutoff level. Second, there is little evidence for negative monotonicity in the evolution of price forecasts. If anything inaccuracy tends to increase over time for markets 2-4. Inexperienced subjects tend to have an increase in forecast accuracy in the first half of the experiment. However the accuracy worsens in the second half of the experiment.\(^{20}\) In summary, there is little evidence for an increase in forecast accuracy over time, suggesting that we can reject hypotheses 3.1 and 3.2.

\(^{20}\)This latter observation for inexperienced subjects is consistent with the view that we observe an increase in fraction of
A reduced form random effects regression – where $\Delta p_t = p_t - p_{t-1}$ of the form:

$$\Delta E^i_{p,t} = \alpha + \alpha_i + \alpha_{sess} + \alpha_2 t + \alpha_3 \Delta p_{t-1} + \epsilon_{i,t}$$  \hspace{1cm} (16)$$

confirms the graphical intuition. The estimation results are shown in Table 1 in columns (1) - (4). Since the estimate for $\alpha_3$ is negative and highly significant for all markets an increase in prices leads to more accurate price forecasts, which could be driven by type transition induced by momentum: A positive price trend could trigger optimism leading to a larger fraction of noise traders which in turn generates prices which are closer to the price prediction of noise traders. However, the reduced form regression only provides suggestive evidence for this reasoning which is the main reason I employ a structural estimation approach further below. Furthermore, at a 5% significance level, the estimate for $\alpha_2$ tends to be either positive or insignificant, suggesting that forecast accuracies tend to decrease and certainly do not increase over time after controlling for price-momentum. Hence the evidence suggests that we can reject any hypothesis which suggests that types homogenize monotonically over time (as in hypotheses 3.1 and 3.2).

In a last step I computed the percentage difference in the price expectations from the asset’s fundamental value; i.e.: the homogeneous belief, rational expectations equilibrium price. I re-estimated the parameters in (16) with this modified measure of expectation-accuracy. The results, which are in line with the results discussed above, are shown in columns (5) - (8) in Table 1.

Overall, the evidence suggests that we can reject hypotheses 3.1 and 3.2: Bubble-crash patterns in the data of Haruvy et al. [2007] are not driven by increasingly rational or increasingly irrational behavior over time.

### Table 1: RE Estimator 1 table

<table>
<thead>
<tr>
<th>(1) (Market 1)</th>
<th>(2) (Market 2)</th>
<th>(3) (Market 3)</th>
<th>(4) (Market 4)</th>
<th>(5) (Market 1)</th>
<th>(6) (Market 2)</th>
<th>(7) (Market 3)</th>
<th>(8) (Market 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_3$</td>
<td>-0.0553</td>
<td>-0.0624</td>
<td>-0.0932</td>
<td>-0.0038</td>
<td>-0.0069</td>
<td>0.0076</td>
<td>-0.00695</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.007)</td>
<td>(0.096)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.103</td>
<td>0.000924</td>
<td>0.0162</td>
<td>0.00347</td>
<td>0.173</td>
<td>0.293</td>
<td>0.0365</td>
</tr>
<tr>
<td>(0.019)</td>
<td>(0.990)</td>
<td>(0.075)</td>
<td>(0.359)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Session Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>$N$</td>
<td>742</td>
<td>795</td>
<td>795</td>
<td>660</td>
<td>742</td>
<td>795</td>
<td>795</td>
</tr>
<tr>
<td>Overall $R^2$</td>
<td>0.218</td>
<td>0.166</td>
<td>0.207</td>
<td>0.105</td>
<td>0.211</td>
<td>0.127</td>
<td>0.062</td>
</tr>
<tr>
<td>$p$-values in parentheses</td>
<td>$^*$ $p &lt; 0.10$, $^{**} p &lt; 0.05$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

4.1 A structural approach

The results from the previous section suggest that there is little evidence for monotonic type-transition. However, there is some evidence that type transition could be triggered by price-momentum as suggested by hypothesis 3.3 (STMD). In this section I will adopt a structural estimation approach to estimate the model’s underlying deep parameters under STMD. I will also show that the model generates an excellent in-sample fit of prices and helps to explain the mechanism through which experience mitigates experimental asset price bubbles.

For estimation purposes I follow the method suggested by Haruvy and Noussair [2006] and Duffy and Ünver [2006]. I denote with $p_t(\sigma^2_\epsilon, \eta, \gamma, \delta, \mu_0, \phi|STMD)$ the average simulated price in period t for a given parameter constellation $(\sigma^2_\epsilon, \eta, \gamma, \delta, \mu_0, \phi)$ and transition dynamics STMD. Along the lines of Haruvy and Noussair [2006] and Duffy and Ünver [2006], I estimate the structural parameters of the model by minimizing noise traders during the bubble phase (which increases forecast accuracy of the average investor). Note that this increase in type-homogeneity would also lead to a decrease in trading volume - a common observation in experimental asset markets during the “bullish phase”. The crash could then be induced even by a few arbitrageurs who discount the asset due to the increase in noise-trader risk induced by the influx of optimistic traders. If the fraction of arbitrageurs, who would be responsible for the crash is sufficiently small the average investor would hold extremely inaccurate price forecasts during the crash - which is again what we observe in Figure 4.
the root-mean-square error (RMSE) between average simulated- and average observed prices, conditional on experience level. To be specific, letting $\bar{p}_t$ be the average observed price in period $t$, I minimize
\[
RMSE = \sqrt{\sum (\bar{p}_t - p_t(\sigma^2, \eta, \gamma, \delta, \mu_0, \phi|STMD))^2},
\] (17)
under STMD, for each experience level separately.\textsuperscript{22}

Following Bossaerts et al. [2007], one can motivate RMSE-minimization in this context, by assuming that arbitrageurs and noise traders make noisy demands, $(\lambda_a^t + \xi_a^t, \lambda_n^t + \xi_n^t)$, whereas the noise terms of arbitrageurs and noise-traders, $(\xi_a^t, \xi_n^t)$, are independently and identically distributed over time, are uncorrelated with dividends and mood-shocks and have means of zero. Under these assumptions, the resulting equilibrium prices take forms like in (11), with the main difference that a random noise term with mean zero is added to them. Minimizing the RMSE from above is then equivalent to minimizing the standard error of this unobservable noise term.\textsuperscript{23}

The fitted price paths from the model and the average observed prices are illustrated in Figures 5a, 5b, 5c and 5d. The dashed lines illustrate the simulated price paths, the solid lines depict the average observed

\textsuperscript{21}Averages are taken across sessions.

\textsuperscript{22}Note that I approximate their uniform distribution of dividends with support $\{0, 4, 14, 30\}$ used by HLN, with a normal distribution with mean $\bar{d}$ and standard deviation of $\sqrt{8}$.

\textsuperscript{23}I employ an interior-point optimization algorithm (Byrd et al. [1999, 2000], Waltz et al. [2006]) to minimize (17).
prices and the straight solid lines show the FV-processes. The estimated parameter values are illustrated in the upper panel in Table 2.

First, the graphical illustration indicates that the model has the potential to explain the bubble-crash patterns observed by HLN and to generate accurate price predictions. Second, the estimated parameter values reveal that the mechanism, through which bubbles are mitigated over experience levels is via a decrease in initial noise-traders \( (\mu_0) \), an increase in forecast accuracies \( (\eta) \) decreases) and faster type transitions \( (\phi) \) increases).\(^{24}\) With the main exception being market 4 mood-volatility, \( \sigma^2 \), decreases as experience increases.

### Table 2: Structural Parameters under STMD

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Market 1</td>
<td>Market 2</td>
<td>Market 3</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>20.5683</td>
<td>20.3558</td>
<td>25.8617</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.7432</td>
<td>0.1153</td>
<td>0.9472</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>0.9884</td>
<td>0.979</td>
<td>0.997</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.9766</td>
<td>0.7864</td>
<td>0.0391</td>
</tr>
<tr>
<td>( \eta )</td>
<td>146.4118</td>
<td>56.0594</td>
<td>36.701</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.0005</td>
<td>0.0772</td>
<td>0.0028</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{24}\)Moreover, the results suggest that risk-aversion levels are not constant across experience levels. As a matter of fact, inexperienced traders tend to be more risk averse than inexperienced traders.

### 4.1.1 Robustness

To check the robustness of the findings, I first use the data-set of Dufwenberg et al. [2005]. Dufwenberg et al. [2005] also consider SSW markets and investigate the impact of experience levels on the formation of bubbles. The authors ran 10 sessions with six subjects each over a ten period trading horizon. Each session consisted of four repeated 10-periods SSW markets. In contrast to HLN, Dufwenberg et al. [2005] use a double auction trading institution and the asset had a dividend support given by \( \{0,20\} \), where each dividend was realized with a probability of 50%. Furthermore, in contrast to HLN, in the fourth repetition of the underlying SSW market only 1/3 or 2/3 of the traders in a market were experienced. The other traders were “removed” and replaced by inexperienced subjects. Since the prices across the 1/3 and 2/3 treatments are not significantly different (Dufwenberg et al. [2005]), I pool the observations over these sessions and estimate the model’s parameters via (17) using the data of Dufwenberg et al. [2005].

The estimated parameter values are illustrated in the mid-panel in Table 2. The estimated (dashed lines) and actual prices (solid lines) are depicted in Figures 6a - 6d. First, focusing on the results in column 1 (market 1) we observe estimates for \( \sigma^2 \), \( \mu_0 \) and \( \delta \) which are fairly consistent with the estimates from the HLN data-set. The fact that the estimate for \( \phi \) is larger in the data-set of Dufwenberg et al. [2005] is generated by the simple fact that their time horizon is shorter, making faster type transition inevitable. The difference in
\( \eta \) is due the fact that the experimental design parameters— in particular in terms of FV— differ substantially between HLN and Dufwenberg et al. [2005]. More specifically, the average FV in Dufwenberg et al. [2005] is 55, whereas it is 96 for HLN generating a ratio of 2.61, which is close to the ratio of estimated \( \eta \)'s (1.75) across the two data-sets. The difference in the estimates for \( \gamma \) for inexperienced subjects between HLN and Dufwenberg et al. [2005] is quite substantial though. The results indicate that mood shocks are less persistent in Dufwenberg et al. [2005], which might be the reason why the underlying average price-dynamics are quite different across the two data-sets.

A quick inspection of the data in Dufwenberg et al. [2005] reveals that the bubble-mitigation effect through experience is less pronounced in comparison to HLN. Hence it is not surprising that \( \mu_0 \) decreases less substantially between markets 1 and 3 (from around 98% to 61%). Most interestingly, the influx of inexperienced traders in market 4 is captured by the estimate for \( \mu_0 \), which again increases as illustrated in column 4. The significant bubble-mitigation effect reported in Dufwenberg et al. [2005] is generated by a significant decrease in \( \sigma_\epsilon^2 \), which parametrizes the volatility in mood-swings of noise traders— another important layer of noise trader risk. The estimates of the other parameters remain fairly stable over the four experience levels in the data of Dufwenberg et al. [2005], which is consistent with the observation that their reported changes in average price dynamics are less pronounced than in the data of HLN.

Lastly, I use the data of Kirchler et al. [2012] (KHS) to estimate the model’s parameters. The experiments of KHS are particularly interesting for the purpose of testing the validity of the model. KHS investigate the effect of confusion on the formation of bubbles in experiments which use the design of SSW. The authors
test whether decreasing fundamental values and/or increasing cash-to-asset (C/A) ratios generate confusion and therefore result into bubbles.

For that purpose the authors ran—among other robustness-check treatments—four treatments each with six sessions consisting of 10 subjects: In treatment 1 the authors used a standard SSW design. In contrast to HLN, the trading horizon of KHS was 10 periods, instead of 15 and the asset could only take on two possible values (0 and 10), each with probability 1/2. Hence, under risk neutrality, homogeneity and rationality, their asset had a fundamental value of 50 in the first period, decreased by a value of five in each of the following periods and finally ended up at a terminal value of 5 in the last period (dividend payments are again deferred to the last period). In treatment 2 the authors used a SSW environment with constant FV and appropriately increasing C/A ratios to mimic the increase in C/A ratios in standard SSW markets. In treatment 3 the authors used a decreasing FV-process but kept the C/A ratio constant, by deducting cash from the subject’s money holdings. In the last treatment the authors kept the FV and the C/A ratio constant.\(^{25}\) The authors observe that constant FV’s mitigate bubbles substantially and conclude that confusion from the decreasing FV generates bubbles in standard SSW markets. I will show that the model presented in this paper will provide even further insights into the underlying bubble-mitigation mechanism observed by KHS.

For that purpose, I re-estimate the structural parameters of the model, using the data of KHS. I illustrate the fitted (dashed lines) and average observed prices from KHS in Figure 7a - 7d. The estimated structural

\(^{25}\)The FV is kept constant by having an asset which pays on average a zero dividend but provides an ensured buyback value in the last period.
parameters are shown in the lower panel of Table 2. Surprisingly, most of the estimated parameters do not change substantially in comparison to the parameters, estimated for inexperienced subjects in the data of Haruvy et al. [2007]. The volatility in the mood-swing shocks ($\sigma^2$), the persistence of mood-swings ($\gamma$) and the initial fraction of noise-traders ($\mu_0$) are remarkably stable, suggesting again that “initial noise” ignites bubbles in experimental asset markets. The value of $\eta$ decreases in the data of KHS. However, as before, one needs to take into account that the fundamental value changed as well, changing the level of prices and therefore also the level of inaccuracy of forecasts.

To be precise, the average fundamental value of the asset in the design of HLN is 96, versus an average FV of 27.5 in Kirchler et al. [2012], giving a ratio of 3.49 between the two average values. Interestingly, the ratio between the two $\eta$'s, estimated for the data of HLN and KHS is 3.989. Hence, after correcting for level-changes, the value of $\eta$ does not change substantially across the two data-sets.

Table 2 shows that $\phi$ increases in the data of KHS, suggesting that types transition faster in Kirchler et al. [2012]. This difference arises again naturally in this context, since the trading horizon of KHS is only 10 periods, whereas the trading horizon is 15 in Haruvy et al. [2007]. The only major difference arises in the estimated risk-aversion parameters. The risk-aversion estimate for the data of KHS is significantly lower than the estimated risk aversion parameters for inexperienced subjects in the data of HLN.

Can the “reduced confusion” argument, provided by KHS, be derived from the estimates across treatments? The answer is yes and moreover the argument can be made more precise. First, in treatment 2, in which the C/A ratio increases and the FV is kept constant the initial fraction of confused traders ($\mu_0$) does not change much in comparison to treatment 1. However, the estimate for the type-transition parameter $\phi$ increases substantially and so does the forecast accuracy of noise-traders ($\eta$ decreases). Moreover the volatility in mood-swings also decreases substantially between treatment 1 and 2, suggesting that constant FV’s alone decreases eradic behavior among noise traders. Nevertheless, the initial fraction of noise-traders does not change much across the two treatments.

Similar (although slightly less pronounced) patterns emerge if we look at the estimates for the parameters in treatments 3 and 4 with one major exception: The estimate for the initial fraction of noise-traders, $\mu_0$, is essentially zero. In both treatments the C/A ratio is kept constant, suggesting that constant C/A-ratios reduce initial confusion or (equivalently) that increasing C/A ratios increase noise-trading. The latter insight is consistent with the results in Caginalp et al. [1998, 2001]. Put differently, constant FV’s reduce the magnitude of confusion among noise traders in the experiments of KHS but they do not suppress initial confusion altogether. Constant C/A ratios on the other hand indeed reduce initial confusion as measured via the fraction of noise traders.

5 Discussion

There are other empirical observations from the experimental asset market literature, which can be explained with the model from above.

Eckel and Füllbrunn [2013] report results that relate experimental asset price bubbles to gender. The authors show that asset price bubbles are mitigated in female subject pools. This evidence can be rationalized with the model above and the insights from Barber and Odean [2001] and (e.g.) Charness and Gneezy [2012]. Barber and Odean [2001] show that male traders are more overconfident than female investors. Charness and Gneezy [2012] provide experimental evidence, suggesting that men are less risk averse than women (measured via $\delta$ in the model above). Along the lines of Kyle and Wang [1997], Odean [1998, 1999], Daniel et al. [1998], Hirshleifer [2001], Daniel et al. [2001] and Scheinkman and Xiong [2003] one can interpret the “bullishness” of noise traders, $\rho_t$, as a measure of overconfidence. Since male traders tend to be more overconfident than female investors one would expect the fraction of noise traders ($\mu$) to be substantially smaller in female subject pools. This potential relationship, together with the observation that female traders tend to be more risk averse than male traders (Charness and Gneezy [2012], Brañas-Garza and Rustichini [2011]), might push down prices in female subject pools and may therefore explain the findings.
of Eckel and Füllbrunn [2013], using the model from above. Baghestanian and Walker [2014] show that trading behavior and visual manipulation of price charts in the first period affect the formation of bubbles in SSW markets. By anchoring prices towards the FV in the first period bubbles essentially vanish. One can rationalize this finding in the model described above via a decrease in $\mu_0$ (initial confusion) and also a decrease in $\eta$ (more accurate price forecasts due to anchoring). Cheung et al. [2014] show that the formation of common expectations before the actual experiment initiates mitigates bubbles in SSW markets. This observation can again be explained via a decrease in $\mu_0$ and $\eta$ as in Baghestanian and Walker [2014].

In summary, the model presented in this paper provides new insights into the sources of bubbles and crashes in experimental asset markets. Price rallies among inexperienced subjects are primarily driven by a large number noise traders. The price rally itself further increases the number of optimistic noise traders via momentum. However, once a certain threshold-level of noise traders is surpassed, their positive price effect is outweighed by their positive impact on the noise-trader-risk-compensation-premium, charged by arbitrageurs. Put differently, if the number of noise traders with excessively optimistic price expectations becomes sufficiently large, the risk that they are subject to negative mood-swings increases as well. Arbitrageurs take this potential risk into account, discount the asset’s value accordingly and induce a crash. The robustness check indicates that the suggested mechanism, which generates bubbles and crashes in the data of Haruvy et al. [2007], is also responsible for similar price patterns observed in the data of Dufwenberg et al. [2005]. Moreover, the results presented in this paper suggest that the model may explain several other empirical observations in the experimental asset market literature.

6 Conclusion

The theoretical and empirical results presented in this paper provide new insights into the mechanisms, responsible for the formation of bubbles and crashes in laboratory environments: First, bubbles are ignited by a large number of noise traders and positive short-term price momentum. The associated price-rally induces a further increase in the fraction of optimistic noise traders, which in turn further feeds the bubble. The upward spiral between optimism and prices comes to an end if the fraction of noise traders reaches a threshold value, after which the noise-trader-risk compensation premium outweighs the positive price effect of noise traders. Once the noise-trader risk compensation premium becomes dominant, prices crash. Experience mitigates bubbles in experimental asset markets through a reduction in noise trader risk. It was shown that the model presented in this paper can also be used to explain various other empirical results presented in the experimental asset market literature.

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26Similarly Oechsler et al. [2011] also show that overconfidence is an important ingredient to bubble formation in experimental asset markets.


Figure 8: Price Data Haruvy et al. [2007]; Solid lines: FV
8 Appendix B

Derivation of (8): First note that
\[ \mathbb{E} d_{t+1} \rho_t = \frac{\eta}{1 - \gamma} \bar{d} \]

Next, \( \mathbb{E} \rho_t (p_{t+1} - p_t) \) can be decomposed into
\[ \mathbb{E} (FV_{t+1} - FV_t) \rho_t = -\frac{\eta}{1 - \gamma} \bar{d} \]

\[ \mathbb{E} (\rho_t (-2\delta(T - (t + 1) + 1) (\sigma_d^2 + \frac{\mu^2 \sigma^2}{(1 - \gamma)^2}) + 2\delta(T - (t + 1) + 1) (\sigma_d^2 + \frac{\mu^2 \sigma^2}{(1 - \gamma)^2}))) = \frac{2\delta \sigma_t^2 (\rho_t + \rho_{t+1})}{1 - \gamma} \]

\[ \mathbb{E} (\rho_t (\mu \eta) \sum_{k=0}^{T-(t+1)} \sum_{j=0}^{k} \gamma^j - \mu \eta \sum_{k=0}^{T-t} \sum_{j=0}^{k} \gamma^j) = -\mu \eta^2 \frac{1 - \gamma^{T-t+1}}{(1 - \gamma)^2} \]

\[ \mathbb{E} (\mu \rho_t \rho_{t+1} + \sum_{j=0}^{T-(t+1)} \gamma^j) = \mu \sum_{j=0}^{T-t} \gamma^j (\frac{\sigma^2}{1 - \gamma^2} + \frac{\eta^2}{(1 - \gamma)^2}) \]

\[ -\mathbb{E} (\mu \rho_t^2 \sum_{j=0}^{T-t} \gamma^j) = -\mu \sum_{j=0}^{T-t} \gamma^j (\frac{\sigma^2}{1 - \gamma^2} + \frac{\eta^2}{(1 - \gamma)^2}) \]

Noting that
\[ \mu \sum_{j=0}^{T-(t+1)} \gamma^j (\frac{\sigma^2}{1 - \gamma^2} + \frac{\eta^2}{(1 - \gamma)^2}) - \mu \sum_{j=0}^{T-t} \gamma^j (\frac{\sigma^2}{1 - \gamma^2} + \frac{\eta^2}{(1 - \gamma)^2}) = -\mu \frac{\sigma^2}{1 - \gamma^2} - \mu \gamma^{T-t} \frac{\eta^2}{(1 - \gamma)^2} \]

and adding terms gives (8).

References


